Decentralized optimization methods for efficient energy management under stochasticity

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Under the supervision of Pierre Carpentier and Michel De Lara

October 25th, 2018

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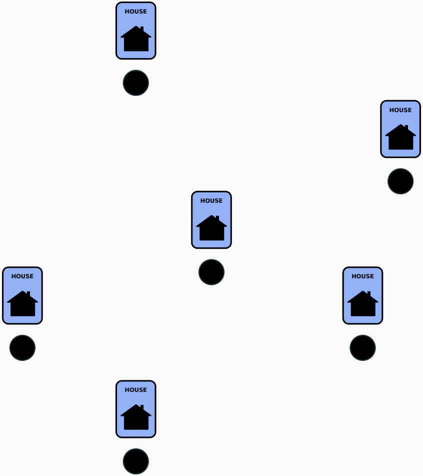
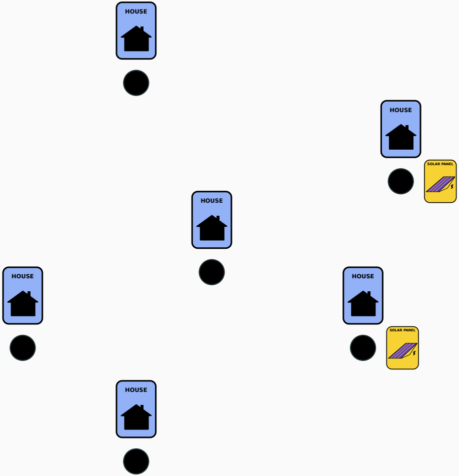
Optimizing energy flows in district



Optimizing energy flows in district



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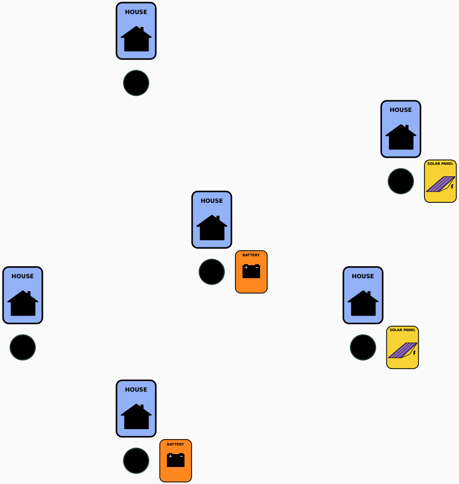
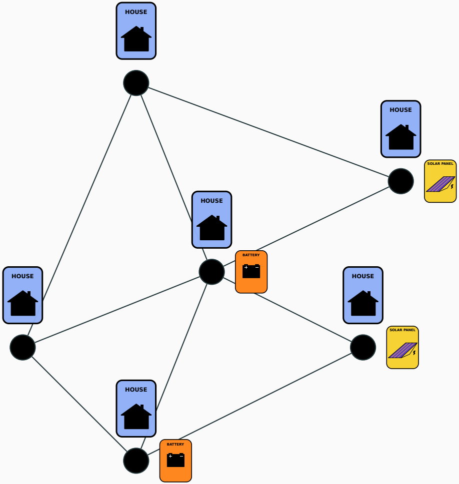


Optimizing energy flows in district



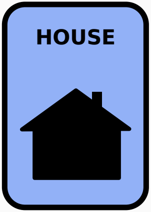
Planning & contributions

1. **Time decomposition in optimization and management of home microgrids**
    - We solve the problem with the algorithm Stochastic Dual Dynamic Programming (SDDP) *(state's dimension = 4)*
    - We compare in a fair manner SDDP with a heuristic policy and with a policy based Model Predictive Control (MPC)

2. **Mixing time and spatial decomposition in large-scale optimization problems**
    - We apply price and resource decompositions to multistage stochastic problem
    - We solve large-scale problems *(with a state dimension up to 64)*
    - We compare decomposition algorithms with the reference SDDP algorithm
    - We show that decomposition algorithms are faster and more accurate than SDDP

Time decomposition in optimization and management of home microgrids

We first study a single building



Let $\{0, 1, \cdots, T-1, T\}$ be a discrete-time span
(here we consider a horizon $T = 24h$ and $\Delta t = 15mn$)

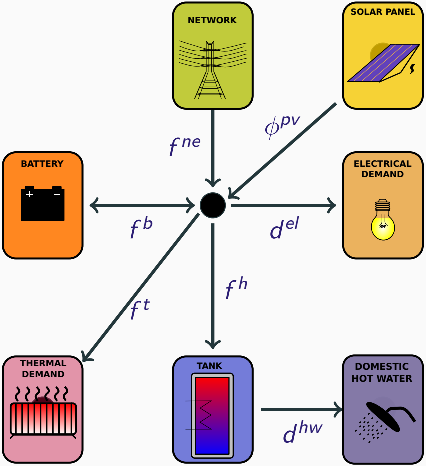
Objective

- We frame a discrete-time optimal control problem
  and consider at all time $t \in \{0, 1, \cdots, T-1\}$
  - An uncertainty $w_t \in \mathbb{W}_t$ *(occuring between $t-1$ and t)*
  - A control $u_t \in \mathbb{U}_t$ *(leveraging the system)*
  - A state $x_t \in \mathbb{X}_t$ *(outlining the energy stocks)*
- We model the uncertainties as random variables
  thus rendering the optimization problem stochastic
- We look at policies

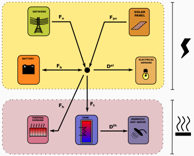
  $$\pi_t : \mathbb{X}_t \to \mathbb{U}_t$$

  to compute decision online for all time $t \in \{0, \cdots, T-1\}$
  (similar approach as in [Bertsekas, 2005]-[Powell, 2014])

We consider the following devices

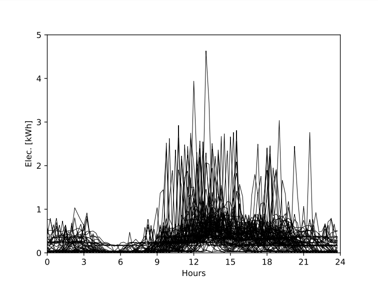


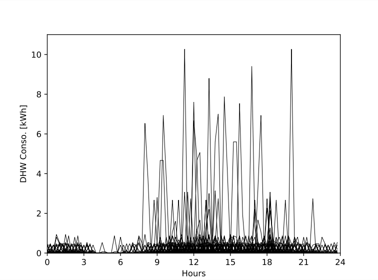
We introduce noises, controls and states



- **Uncertainties** $\mathbf{W}_t = \left(\mathbf{D}^{el}_t, \mathbf{D}^{hw}_t\right)$
  - $\mathbf{D}^{el}_t$, electrical demand (kW)
  - $\mathbf{D}^{hw}_t$, domestic hot water demand (kW)
- **Control variables** $\mathbf{U}_t = \left(\mathbf{F}_{\mathbf{B},t}, \mathbf{F}_{T,t}, \mathbf{F}_{\mathbf{H},t}\right)$
  - $\mathbf{F}_{\mathbf{B},t}$, energy exchange with the battery (kW)
  - $\mathbf{F}_{T,t}$, energy used to heat the hot water tank (kW)
  - $\mathbf{F}_{\mathbf{H},t}$, thermal heating (kW)
- **Stock variables** $\mathbf{X}_t = \left(\mathbf{B}_t, \mathbf{H}_t, \boldsymbol{\theta}^i_t, \boldsymbol{\theta}^w_t\right)$
  - $\mathbf{B}_t$, battery level (kWh)
  - $\mathbf{H}_t$, hot water storage (kWh)
  - $\boldsymbol{\theta}^i_t$, inner temperature ($^\circ C$)
  - $\boldsymbol{\theta}^w_t$, wall's temperature ($^\circ C$)

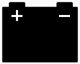
Looking at scenarios of demands during one day





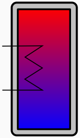
Discrete time state equations

We have the four state equations (all linear), describing the evolution over time of the stocks:



$$\mathbf{B}_{t+1} = \alpha_{\mathbf{B}} \mathbf{B}_t + \Delta T \Big(\rho_c \mathbf{F}^{+}_{\mathbf{B},t} - \frac{1}{\rho_d} \mathbf{F}^{-}_{\mathbf{B},t} \Big)$$

$$\mathbf{H}_{t+1} = \alpha_{\mathbf{H}} \mathbf{H}_t + \Delta T \big[\mathbf{F}_{T,t} - \mathbf{D}^{hw}_{t+1} \big]$$



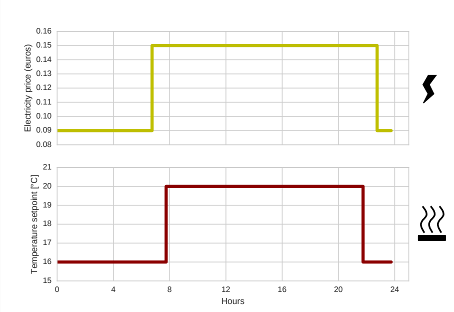
$$\boldsymbol{\theta}^{w}_{t+1} = \boldsymbol{\theta}^{w}_{t} + \frac{\Delta T}{c_m} \left[\frac{\boldsymbol{\theta}^{i}_{t} - \boldsymbol{\theta}^{w}_{t}}{R_i + R_s} + \frac{\boldsymbol{\theta}^{e}_{t} - \boldsymbol{\theta}^{w}_{t}}{R_m + R_e} + \gamma \mathbf{F}_{\mathbf{H},t} + \frac{R_i}{R_i + R_s} P^{int}_t + \frac{R_e}{R_e + R_m} P^{ext}_t \right]$$

$$\boldsymbol{\theta}^{i}_{t+1} = \boldsymbol{\theta}^{i}_{t} + \frac{\Delta T}{c_i} \left[\frac{\boldsymbol{\theta}^{w}_{t} - \boldsymbol{\theta}^{i}_{t}}{R_i + R_s} + \frac{\boldsymbol{\theta}^{e}_{t} - \boldsymbol{\theta}^{i}_{t}}{R_v} + \frac{\boldsymbol{\theta}^{e}_{t} - \boldsymbol{\theta}^{i}_{t}}{R_f} + (1-\gamma) \mathbf{F}_{\mathbf{H},t} + \frac{R_s}{R_i + R_s} P^{int}_t \right]$$

which will be denoted

$$\boxed{\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1})}$$

Prices and temperature setpoints vary along time



- $T = 24$h, $\Delta T = 15$mn
- Electricity peak and off-peak hours
  $p_t^E = 0.09$ or 0.15 euros/kWh
- Temperature set-point
  $\bar{\theta}_t^i = 16^\circ C$ or $20^\circ C$

The costs we have to pay

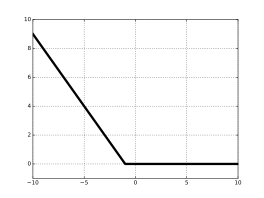
- Cost to import electricity from the network

$$p_t^E \times \max\{0, \mathbf{F}_{NE,t+1}\}$$

where we define the recourse variable (electricity balance):

$$\underbrace{\mathbf{F}_{NE,t+1}}_{Network} = \underbrace{\mathbf{D}^{el}_{t+1}}_{Demand} + \underbrace{\mathbf{F}_{\mathbf{B},t}}_{Battery} + \underbrace{\mathbf{F}_{\mathbf{H},t}}_{Heating} + \underbrace{\mathbf{F}_{T,t}}_{Tank} - \underbrace{\mathbf{F}_{pv,t}}_{Solar\ \ panel}$$

- Virtual Cost of thermal discomfort: $\kappa_{th}\Big(\underbrace{\boldsymbol{\theta}^i_t - \bar{\boldsymbol{\theta}}^i_t}_{\text{deviation from setpoint}} \Big)$



κ_{th}
Piecewise linear cost which penalizes temperature if below given setpoint

Instantaneous and final costs for a single house

- The instantaneous convex costs are

$$L_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1}) = \underbrace{p_t^E \max\{0, \mathbf{F}_{NE,t+1}\}}_{bill} + \underbrace{\kappa_{th}(\theta_t^i - \bar{\theta}_t^i)}_{discomfort}$$

- We add a final linear cost

$$K(\mathbf{X}_T) = -p^{\mathbf{H}}\mathbf{H}_T - p^{\mathbf{B}}\mathbf{B}_T$$

to avoid empty stocks at the final horizon T

Writing the stochastic optimization problem

We now write the optimization problem

$$\min_{\mathbf{X},\mathbf{U}} \mathbb{E}\left[\sum_{t=0}^{T-1} L_t(\mathbf{X}_t,\mathbf{U}_t,\mathbf{W}_{t+1}) + K(\mathbf{X}_T)\right]$$

$$\text{s.t. } \mathbf{X}_{t+1} = f_t(\mathbf{X}_t,\mathbf{U}_t,\mathbf{W}_{t+1}), \quad \mathbf{X}_0 = x_0 \quad \text{(Dynamic)}$$

$$x^\flat \leq \mathbf{X}_t \leq x^\sharp \quad \text{(Bounds)}$$

$$\sigma(\mathbf{U}_t) \subset \sigma(\mathbf{W}_1,\ldots,\mathbf{W}_t) \quad \text{(Non-anticipativity)}$$

We aim at minimizing the expected value of the sum of the operational costs

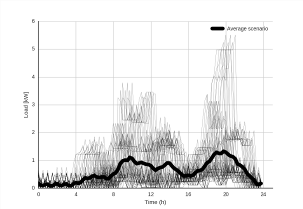
We look at solution as state feedback policies $\pi_t : \mathbb{X}_t \to \mathbb{U}_t$

$$\mathbf{U}_t = \pi_t(\mathbf{X}_t)$$

Time decomposition in optimization and management of home microgrids

Numerical results

Model Predictive Control (MPC)



Procedure

Input

- A deterministic forecast scenario $(\overline{w}_{t+1}, \cdots, \overline{w}_T)$ (possibly updated)

Output

- A policy $\pi_t^{mpc} : \mathbb{X}_t \to \mathbb{U}_t$ that maps the current state x_t to a decision u_t

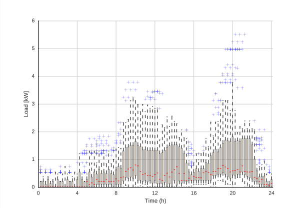
The MPC policy $\pi_t^{mpc} : \mathbb{X}_t \to \mathbb{U}_t$ writes, for all time $t \in \{0, \cdots, T-1\}$

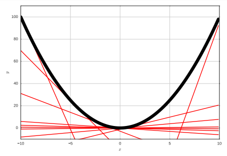
$$\pi_t^{mpc}(x_t) \in \arg\min_{x,u} \sum_{s=t}^{T-1} L_s(x_s, u_s, \overline{w}_{s+1}) + K(x_T)$$

$$\text{s.t. } x_{s+1} = f_s(x_s, u_s, \overline{w}_{s+1})$$

and corresponds to solve a deterministic optimization problem

Stochastic Dual Dynamic Programming





Procedure [Pereira and Pinto, 1991]

Input

- A family of discrete marginal distributions $\mu_{t+1}(\cdot) = \sum_{s=1}^{S} \pi_s \delta_{w_{t+1}^s}(\cdot)$ with $\sum_{s=1}^{S} \pi_s = 1$

Output

- Value functions $\underline{V}_t : \mathbb{X}_t \to \mathbb{R}$ approximating the original Bellman functions $V_t : \mathbb{X}_t \to \mathbb{R}$ as a supremum of affine functions

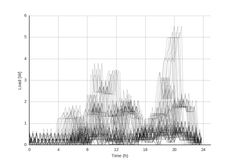
$$\underline{V}_t(x) = \max_{1 \le k \le K} \{\lambda_t^k x + \beta_t^k\} \le V_t(x)$$

- A policy $\pi_t^{sddp} : \mathbb{X}_t \to \mathbb{U}_t$ that maps the current state x_t to a decision u_t

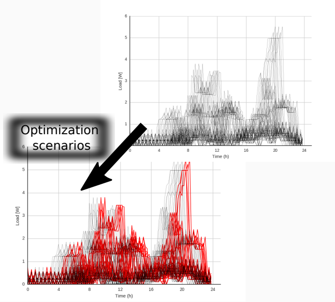
The SDDP policy $\pi^{sddp} : \mathbb{X}_t \to \mathbb{U}_t$ writes, for all time $t \in \{0, \cdots, T-1\}$

$$\pi_t^{sddp}(x_t) \in \arg\min_{u_t \in \mathbb{U}_t} \sum_{s=1}^{S} \pi_s \big[L_t(x_t, u_t, w_{t+1}^s) + \underline{V}_{t+1}\big(f_t(x_t, u_t, w_{t+1}^s)\big)\big]$$

How to assess MPC and SDDP strategies?



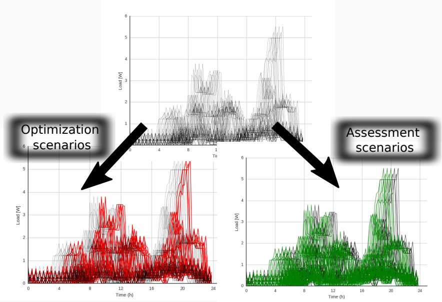
How to assess MPC and SDDP strategies?



Optimization scenarios

- Dedicated to fit probability laws or forecasts
- All algorithms have access to the same set of scenarios
- Algorithms do not have access to assessment scenarios

How to assess MPC and SDDP strategies?



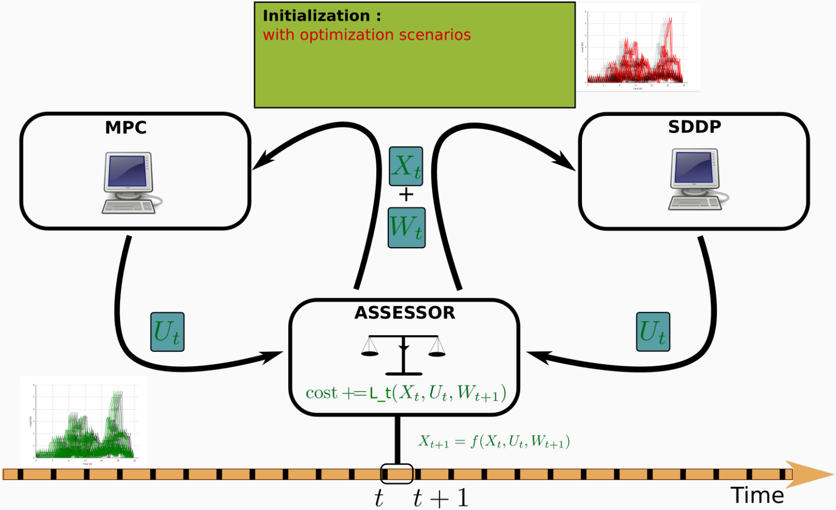
Optimization scenarios

- Dedicated to fit probability laws or forecasts
- All algorithms have access to the same set of scenarios
- Algorithms do not have access to assessment scenarios

Assessment scenarios

- Dedicated to assess the performance of the different policies
- We simulate each policy along each assessment

Assessment procedure



Comparison of MPC and SDDP

We compare MPC and SDDP over 1,000 assessment scenarios

	SDDP	MPC	Heuristic
Electricity bill (€)			
Winter day	4.38 $\pm$ 0.02	4.59 $\pm$ 0.02	5.55 $\pm$ 0.02
Spring day	1.46 $\pm$ 0.01	1.45 $\pm$ 0.01	2.83 $\pm$ 0.01
Summer day	0.10 $\pm$ 0.01	0.18 $\pm$ 0.01	0.33 $\pm$ 0.02

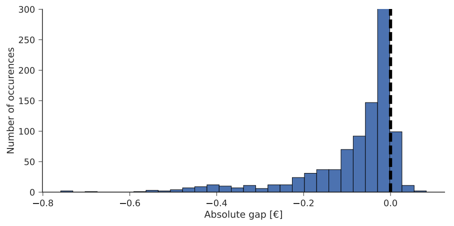


Figure 1: Absolute gap savings between MPC and SDDP during *Summer day*

Conclusion for the single house problem

Contributions

- We begin the study by a simple example
- We have formulated a stochastic optimization problem for domestic energy management system
- We have compared two resolution algorithms (MPC and SDDP)
- On this particular example, SDDP gives better performance than MPC

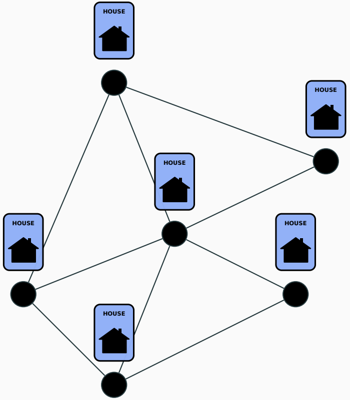
Perspectives

- Compare SDDP with a stochastic version of MPC (based on scenario trees)
- Extension to larger problems (curse of dimensionality): in the following!

Mixing time and spatial decomposition in large-scale optimization problems

The challenge is now to be able to tackle larger problems

We now consider a *peer-to-peer* community,
where different buildings exchange energy



Objective

- We will formulate a large scale (stochastic) optimization problem
- We will apply decomposition algorithm on it
- We introduce a new formalism that generalizes the algorithm *Dual Approximate Dynamic Programming* ([Girardeau, 2010][Leclère, 2014])

Mixing time and spatial decomposition in large-scale optimization problems

Optimization upper and lower bounds by decomposition

Decompose optimization problem with coupling constraints

Let, for $i \in \{1, \cdots, N\}$

- $\mathcal{C}^i$ be a Hilbert space
- $u^i \in \mathbb{U}^i$ be a decision variable
- $J^i : \mathbb{U}^i \to \mathbb{R}$ be a local objective
- $\Theta^i : \mathbb{U}^i \to \mathcal{C}^i$ be a mapping
- $S \subset \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$ be a set

We consider the following problem

$$V^\sharp = \inf_{u^1, \cdots, u^N} \sum_{i=1}^{N} J^i(u^i)$$

$$\text{s.t. } \underbrace{\big(\Theta^1(u^1), \cdots, \Theta^N(u^N)\big) \in S}_{\text{coupling constraint}}$$

Price and resource value functions provide bounds

We define for $i \in \{1, \cdots, N\}$

- The *local price value function*

$$\underline{V}^i[\lambda^i] = \min_{u^i} \; J^i(u^i) + \left\langle \lambda^i \,, \Theta^i(u^i) \right\rangle \,, \;\; \forall \lambda^i \in (\mathcal{C}^i)^\star$$

- The *local resource value function*

$$\overline{V}^i[r^i] = \min_{u^i} J^i(u^i) \,, \;\; \text{s.t. } \Theta^i(u^i) = r^i \,, \qquad \forall r^i \in \mathcal{C}^i$$

Theorem

For any

- *admissible price* $\lambda = (\lambda^1, \cdots, \lambda^N) \in S^o = \left\{ \lambda \in \mathcal{C}^\star \mid \left\langle \lambda \,, r \right\rangle \leq 0 \,, \;\; \forall r \in \mathcal{S} \right\}$
- *admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^N \underline{V}^i[\lambda^i] \leq V^\sharp \leq \sum_{i=1}^N \overline{V}^i[r^i]$$

Application to stochastic optimal control

We now consider the stochastic optimal control problem

$$V_0^\sharp(x_0) = \min_{\mathbf{X},\mathbf{U}} \mathbb{E}\Big[\sum_{i=1}^{N}\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i)\Big]$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})\ , \quad \mathbf{X}_0^i = x_0^i$$

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_t)$$

$$\big(\Theta_t^1(\mathbf{X}_t^1, \mathbf{U}_t^1), \cdots, \Theta_t^N(\mathbf{X}_t^N, \mathbf{U}_t^N)\big) \in S_t$$

with

- $\mathbf{W} = (\mathbf{W}_0, \cdots, \mathbf{W}_T)$ a global white noise process
- $\mathbf{U} = (\mathbf{U}_0^i, \cdots, \mathbf{U}_{T-1}^i)$ a local control process
- $\mathbf{X}^i = (\mathbf{X}_0^i, \cdots, \mathbf{X}_T^i)$ a local state process
- $g_t^i : \mathbb{X}_t^i \times \mathbb{U}_t^i \times \mathbb{W}_{t+1} \to \mathbb{X}_{t+1}^i$ a local dynamics
- $L_t^i : \mathbb{X}_t^i \times \mathbb{U}_t^i \times \mathbb{W}_{t+1} \to \mathbb{R}$ a local instantaneous cost
- $K^i : \mathbb{X}_T^i \to \mathbb{R}$ a local final cost
- $\Theta_t^i : \mathbb{X}_t^i \times \mathbb{U}_t^i \to \mathcal{C}^i$ a local coupling

Obtaining bounds for the global problem

Theorem

For any

- *admissible price process* $\boldsymbol{\lambda} = (\boldsymbol{\lambda}^1, \cdots, \boldsymbol{\lambda}^N) \in S^o$
- *admissible resource process* $\mathbf{R} = (\mathbf{R}^1, \cdots, \mathbf{R}^N) \in S$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{\lambda}^i](x_0^i) \leq V_0(x_0) \leq \sum_{i=1}^{N} \overline{V}_0^i[\mathbf{R}^i](x_0^i)$$

Price local value function

$$\underline{V}_0^i[\boldsymbol{\lambda}^i](x_0^i) = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \lambda_t^i, \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \rangle + K^i(\mathbf{X}_T^i)\Big]$$
$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i$$
$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_t)$$

Resource local value function

$$\overline{V}_0^i[\mathbf{R}^i](x_0^i) = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i)\Big]$$
$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i$$
$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_t), \quad \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) = \mathbf{R}_t^i$$

Mixing price/resource and temporal decompositions

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{\lambda}^i](x_0^i) \leq V_0(x_0) \leq \sum_{i=1}^{N} \overline{V}_0^i[\mathbf{R}^i](x_0^i)$$

Price decomposition

- Fix a deterministic price $\lambda = (\lambda^1, \cdots, \lambda^N) \in S^o$
- Obtain $\underline{V}_0^i[\lambda^i](x_0^i)$ by Dynamic Programming with local state x_t^i

$$\underline{V}_t^i(x_t^i) = \min_{u_t^i} \mathbb{E}\big[L_t(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \langle \lambda_t^i , \Theta_t^i(x_t^i, u_t^i) \rangle + \underline{V}_{t+1}^i(g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1})\big]$$

- Return the value functions $\{\underline{V}_t^i\}$

Resource decomposition

- Fix a deterministic resource $r = (r^1, \cdots, r^N) \in S$
- Obtain $\overline{V}_0^i[r^i](x_0^i)$ by Dynamic Programming with local state x_t^i

$$\overline{V}_t^i(x_t^i) = \min_{u_t^i} \mathbb{E}\big[L_t(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \overline{V}_{t+1}^i(g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1})\big] \quad \text{s.t. } \Theta_t^i(x_t^i, u_t^i) = r_t^i$$

- Return the value functions $\{\overline{V}_t^i\}$

Tightening the bounds in the inequalities

Looking at optimal coordination processes

- We look at deterministic coordination processes
  to solve the subsystems locally by Dynamic Programming
- The inequalities holds if we look at optimal coordination processes

$$\max_{(\lambda^1,\cdots,\lambda^N)\in S^o} \sum_{i=1}^{N} \underline{V}_0^i[\lambda^i](x_0^i) \leq V_0(x_0) \leq \min_{(r^1,\cdots,r^N)\in S} \sum_{i=1}^{N} \overline{V}_0^i[r^i](x_0^i)$$

Deducing two control policies

Once value functions $\underline{V}_t^i$ and $\overline{V}_t^i$ computed, we define

- the global price policy

$$\underline{\pi}_t(x_t^1, \cdots, x_t^N) \in \mathop{\arg\min}_{u_t^1, \cdots, u_t^N} \mathbb{E}\Big[\sum_{i=1}^N L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \underline{V}_{t+1}^i(\mathbf{X}_{t+1}^i)\Big]$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) \ , \ \ \forall i \in \{1, \cdots, N\}$$

$$\big(\Theta_t(x_t^1, u_t^1), \cdots, \Theta_t(x_t^N, u_t^N)\big) \in S_t$$

- the global resource policy

$$\overline{\pi}_t(x_t^1, \cdots, x_t^N) \in \mathop{\arg\min}_{u_t^1, \cdots, u_t^N} \mathbb{E}\Big[\sum_{i=1}^N L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \overline{V}_{t+1}^i(\mathbf{X}_{t+1}^i)\Big]$$

$$\text{s.t. } \mathbf{X}_{t+1}^i = g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) \ , \ \ \forall i \in \{1, \cdots, N\}$$

$$\big(\Theta_t(x_t^1, u_t^1), \cdots, \Theta_t(x_t^N, u_t^N)\big) \in S_t$$

Where are we heading to?

- First, we have obtained upper and lower bounds
  for global optimization problems with coupling constraints
  thanks to two spatial decomposition schemes
  - Price decomposition
  - Resource decomposition

- Second, with proper coordinating price and resource processes
  we have computed the lower and upper bounds
  by Dynamic Programming (temporal decomposition)

- With the upper and lower Bellman value functions,
  we have deduced two online policies

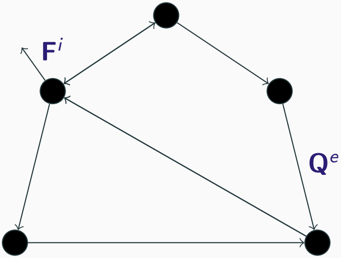
- Now, we will apply these decomposition schemes
  to large-scale problems

Mixing time and spatial decomposition in large-scale optimization problems

Nodal decomposition of a network optimization problem

Modeling flows between nodes

Graph $G = (\mathcal{V}, \mathcal{E})$



- $\mathbf{Q}_t^e$ flow through edge e,
- $\mathbf{F}_t^i$ flow imported at node i

Let A be the *node-edge* incidence matrix

At each time $t \in \{0, \cdots, T-1\}$, Kirchhoff current law couples nodal and edge flows

$$A\mathbf{Q}_t + \mathbf{F}_t = 0$$

Writing down the local problem at node i

We aim at minimizing the nodal costs over the nodes $i \in \mathcal{V}$

$$J^i_{\mathcal{V}}(\mathbf{F}^i) = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} \underbrace{L^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{W}_{t+1})}_{\text{instantaneous cost}} + K^i(\mathbf{X}^i_T) \Big]$$

subject to, for all $t \in \{0, \cdots, T-1\}$

i) The nodal dynamics constraint (for battery and hot water tank)

$$\mathbf{X}^i_{t+1} = g^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{W}_{t+1})$$

ii) The non-anticipativity constraint (future remains unknown)

$$\sigma(\mathbf{U}^i_t) \subset \sigma(\mathbf{W}_0, \cdots, \mathbf{W}_{t+1})$$

iii) The load balance equation (production + import = demand)

$$\Delta^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) = 0$$

Transportation costs are decoupled in time

At each time step $t \in \{0, \cdots, T-1\}$, we define the edges cost as the sum of the costs of flows $\mathbf{Q}_t^e$ through the edges e of the grid

$$J_{\mathcal{E},t}(\mathbf{Q}_t) = \mathbb{E}\Big(\sum_{e \in \mathcal{E}} l_t^e(\mathbf{Q}_t^e)\Big)$$

Global optimization problem

The *nodal cost* $J_{\mathcal{V}}$ aggregates the costs at all nodes i

$$J_{\mathcal{V}}(\mathbf{F}) = \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(\mathbf{F}^i)$$

and the *edge cost* $J_{\mathcal{E}}$ aggregates the edges costs at all time t

$$J_{\mathcal{E}}(\mathbf{Q}) = \sum_{t=0}^{T-1} J_{\mathcal{E},t}(\mathbf{Q}_t)$$

The global optimization problem writes

$$V^\sharp = \min_{\mathbf{F},\mathbf{Q}} J_{\mathcal{V}}(\mathbf{F}) + J_{\mathcal{E}}(\mathbf{Q})$$
$$\text{s.t. } A\mathbf{Q} + \mathbf{F} = 0$$

What do we plan to do?

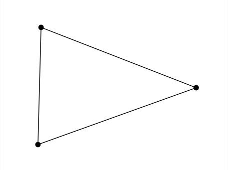
- We have formulated a multistage stochastic optimization problem on a graph
- We will handle the coupling Kirchhoff constraints by the two methods presented earlier
  - Price decomposition
  - Resource decomposition
- We will show the scalability of decomposition algorithms (we solve problems with up to 48 buildings)

Mixing time and spatial decomposition in large-scale optimization problems

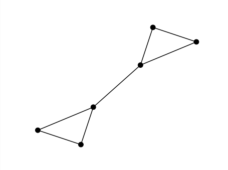
Numerical results on urban microgrids

We consider different urban configurations

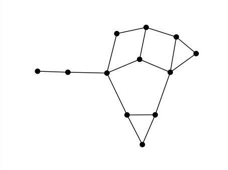
3-Nodes

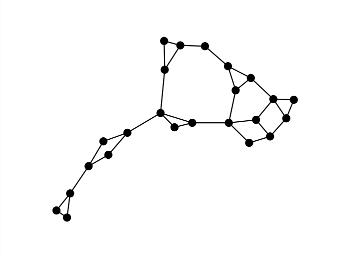


6-Nodes

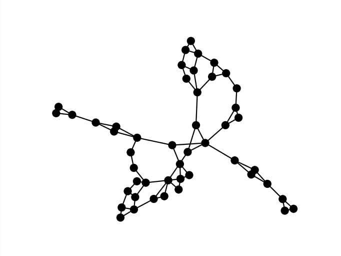


12-Nodes





24-Nodes



48-Nodes

Problem settings

- One day horizon at 15mn time step: $T = 96$

- Weather corresponds to a sunny day in Paris *(June 28th, 2015)*

- We mix three kinds of buildings
  1. Battery + Electrical Hot Water Tank
  2. Solar Panel + Electrical Hot Water Tank
  3. Electrical Hot Water Tank

  and suppose that all consumers are commoners sharing their devices

Looking for appropriate price and resource processes

We define the price function as

$$\underline{V}[\boldsymbol{\lambda}] = \min_{\mathbf{F},\mathbf{Q}} \; J_P(\mathbf{F}) + J_T(\mathbf{Q}) + \left\langle \boldsymbol{\lambda} \,, A\mathbf{Q} + \mathbf{F} \right\rangle$$

and the resource function as

$$\overline{V}[\mathbf{R}] = \min_{\mathbf{F},\mathbf{Q}} J_P(\mathbf{F}) + J_T(\mathbf{Q})$$
$$\text{s.t. } \mathbf{F} = A\mathbf{R} \,, \quad \mathbf{Q} = -\mathbf{R}$$

Objective

We aim to find deterministic price λ and resource sequences r that tighten the gap

$$\max_{\boldsymbol{\lambda} \text{ det}} \; \underline{V}[\boldsymbol{\lambda}] \leq V^\sharp \leq \min_{\mathbf{R} \text{ det}} \; \overline{V}[\mathbf{R}]$$

Algorithms inventory

Nodal decomposition

- Encompass price and resource decompositions
- Resolution by Quasi-Newton (BFGS) methods

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}(\boldsymbol{\lambda}^{(k)})$$

- BFGS iterates till no descent direction is found
- Each nodal subproblem solved by local SDDP (quickly converges)
- Oracle $\nabla \underline{V}(\boldsymbol{\lambda}) = \mathbb{E}[A\mathbf{Q}^\sharp(\boldsymbol{\lambda}) + \mathbf{F}^\sharp(\boldsymbol{\lambda})]$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Global SDDP

We use as a reference the SDDP algorithm applied globally

- Noises $\mathbf{W}_t^1, \cdots, \mathbf{W}_t^N$ are independent node by node (total support size is $|supp(\mathbf{W}_t^i)|^N$). Need to resample the support!
- Level-one cut selection algorithm (keep 100 most relevant cuts)
- Converged once gap between UB and LB is lower than 1%

Fortunately, everything converges nicely! For Global SDDP...

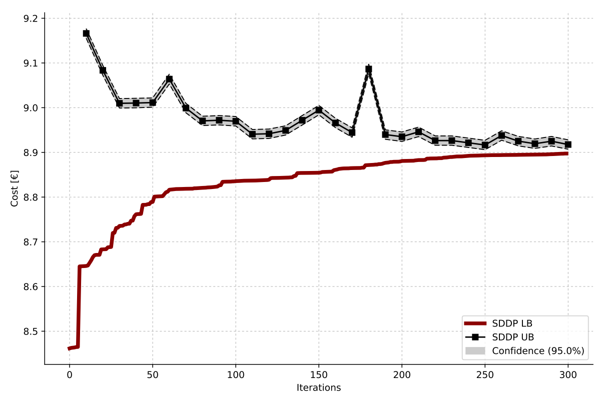


Figure 2: Global SDDP convergence (upper and lower bounds)

...and for nodal decomposition

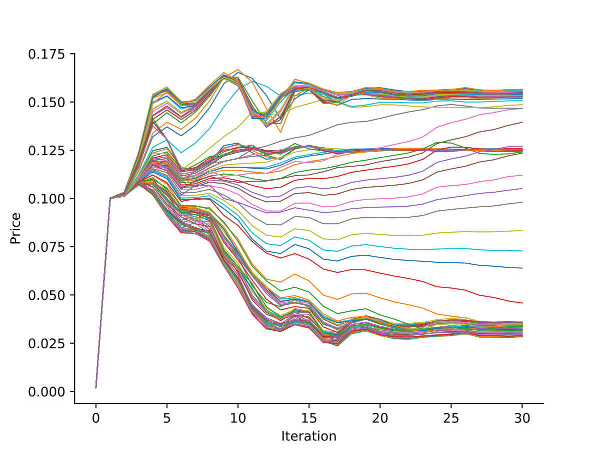


Figure 3: We display the evolution along iterations of the price vector $(\lambda_0^1, \cdots, \lambda_{T-1}^1)$ corresponding to **Node 1**

Upper and lower bounds on the global problem

	Graph	**3-Nodes**	**6-Nodes**	**12-Nodes**	**24-Nodes**	**48-Nodes**
State dim.	$\lvert\mathbb{X}\rvert$	4	8	16	32	64
Global SDDP	time	1'	3'	10'	79'	453'
Global SDDP	LB	2.252	4.559	8.897	17.528	33.103
Price	time	6'	14'	29'	41'	128'
Price	LB	2.137	4.473	8.967	17.870	33.964
Resource	time	3'	7'	22'	49'	91'
Resource	UB	2.539	5.273	10.537	21.054	40.166

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- For the **48-Nodes** problem

$$\begin{array}{ccccccc} \underline{V}_0[sddp] & \leq & \underline{V}_0[price] & \leq & V^\sharp & \leq & \overline{V}_0[resource] \\ 33.103 & \leq & 33.964 & \leq & V^\sharp & \leq & 40.166 \end{array}$$

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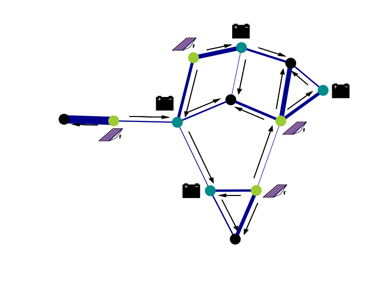
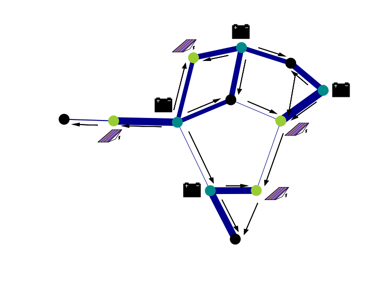
- For the **48-Nodes** problem, Price Decomposition is almost 3x as fast as Global SDDP (and parallelization is straightforward!)

Optimal flows in simulation for 12-Nodes problem

1. We simulate Price policy over 1,000 scenarios
2. We look at flows at two moments in the day



12pm



9pm

Policy evaluation by Monte Carlo simulation

Graph	**3-Nodes**	**6-Nodes**	**12-Nodes**	**24-Nodes**	**48-Nodes**
SDDP policy	2.26 ± 0.006	4.71 ± 0.008	9.36 ± 0.011	18.59 ± 0.016	35.50 ± 0.023
Price policy	2.28 ± 0.006	4.64 ± 0.008	9.23 ± 0.012	18.39 ± 0.016	34.90 ± 0.023
Gap	-0.9 %	+1.5%	+1.4%	+1.1%	+1.7%
Resource policy	2.29 ± 0.006	4.71 ± 0.008	9.31 ± 0.011	18.56 ± 0.016	35.03 ± 0.022
Gap	-1.3 %	0.0%	+0.5%	+0.2%	+1.2%

Price policy beats numerically Global SDDP policy and resource policy

For the **48-Nodes** problem:

$$
\begin{array}{ccccccc}
V^\sharp & \leq & C[price] & \leq & C[resource] & \leq & C[sddp] \\
V^\sharp & \leq & 34.90 & \leq & 35.03 & \leq & 35.50
\end{array}
$$

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V^\sharp & \leq & 34.90 & \leq & 35.03 & \leq & 35.50
\end{array}
$$

We observe that

$$
\begin{array}{ccccc}
V^\sharp & \leq & C[price] & \leq & \overline{V}_0[resource] \\
V^\sharp & < & 34.9 & < & 40.2
\end{array}
$$

From deterministic to Markovian processes

Extension to Markovian processes [Alais, 2013]

- We are also able to consider Markovian coordination processes
- We consider
$$\boldsymbol{\lambda}_t = \phi_t(\mathbf{Y}_t)$$
where $(\mathbf{Y}_0, \cdots, \mathbf{Y}_T)$ is a Markovian process satisfying the dynamics
$$\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$$
- We easily adapt the local DP equations to the Markovian case with the extended state (x_t^i, y_t)

$$\underline{V}_t^i(x_t^i, \underline{y}_t) = \min_{u_t^i \in \mathbb{U}_t^i} \mathbb{E}\Big[L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}^i) + \phi_t^i(\underline{y}_t) \cdot \Theta_t^i(x_t^i, u_t^i) + \underline{V}_{t+1}^i\big(g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}^i), h_t(\underline{y}_t, \mathbf{W}_{t+1})\big)\Big]$$

Conclusion

- We have presented two algorithms that decompose, spatially then temporally, a global optimization problem under coupling constraints

- On this case study, decomposition beat global SDDP for large instances ($\geq$ 24 nodes)
  - In running time (3.5x faster for **48-Nodes**)
  - In precision ($>$ 1% better)

- Can we obtain tighter bounds?
  If we select properly the resource and price processes $\mathbf{R}$ and $\boldsymbol{\lambda}$, among Markovian ones (instead of deterministic ones) we can obtain nodal value functions — with an extended local state

Conclusion

Contributions & Perspectives

Contributions

- We have formulated energy management systems as a stochastic optimization problem and compare different policies
- We have designed two decomposition algorithms to tackle large-scale problems and applied them to damsvalleys and urban microgrids
- We have improved SDDP by allowing to compute a deterministic upper bound (by exploiting Fenchel duality)

Perspectives

- Are decomposition algorithms as effective for problems with stronger connections between subproblems?
- Does using Markovian resource process improve the performance of resource decomposition?
- Is it possible to use more complicated decomposition schemes (by prediction, operator splitting methods...)?

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