Optimization of Energy Production and Transport

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Approaches by Decomposition

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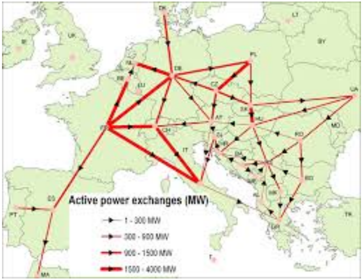
Motivation

An energy production and transport optimization problem on a grid modeling energy exchange across countries.[2]



- Stochastic dynamical problem.
- Discrete time formulation (one-day time step).
- Large-scale problem (many countries).

[2] But the framework remains valid for smaller energy management problems.

Goal

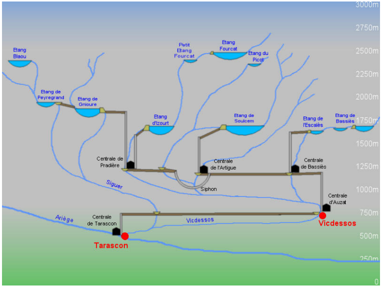
Obtain cost-to-go functions for a large scale stochastic optimal control problem in discrete time.

- In order to obtain decision strategies (closed-loop controls), we have to use dynamic programming or related methods.
  - **Assumption**: Markovian case,
  - **Difficulty**: curse of dimensionality.
- To overcome the barrier of the dimension we want to use decomposition/coordination techniques, which makes it difficult to take into account the information pattern induced by the stochasticity in the optimization problem.

This is a part of a broader project, aiming to develop decision analysis tools for long-term investment problems.

Previous work

We studied the application of stochastic decomposition to the optimization of an hydraulic valley.



Valley: a tree structure with

- nodes: dams
- arcs: flows

We solved this problem using a price-decomposition approach (see [Carpentier et al, 2017]).

We want to extend this work in two directions:

- more complex topologies (graphs rather than trees)
- other decomposition algorithms (allocation, prediction).

Lecture outline

1. Introduction
   - The production and transport problem
   - Mixing decomposition and dynamic programming
2. Decomposition methods
   - Price decomposition
   - Resource allocation
   - Interaction prediction
3. Discussion

1. Introduction
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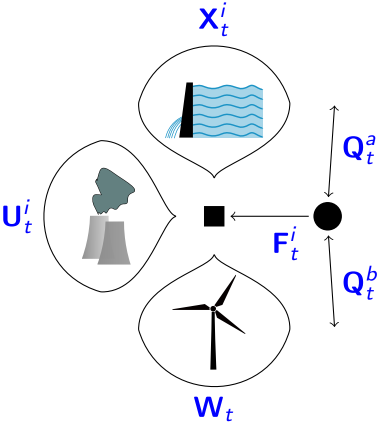
Production at each node of the grids

At each node i of the grid, we formulate a production problem on a discrete time horizon $\mathrm{J}0, T\mathrm{K}$, involving at each time t the following variables:



- $\mathbf{X}_t^i$: state variable (dam level)
- $\mathbf{U}_t^i$: control variable (units production)
- $\mathbf{F}_t^i$: grid flow (import/export from the grid)
- $\mathbf{W}_t$: noise (consumption, renewable)

The noise $\mathbf{W}_t$ is supposed to be shared across the different nodes.

A stochastic optimization problem decoupled in space

At each node i of the grid, we have to solve a stochastic optimal control subproblem depending on the grid flow process $\mathbf{F}^i$:

$$J^i_{\mathfrak{P}}[\mathbf{F}^i] = \min_{\mathbf{X}^i, \mathbf{U}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) + K^i(\mathbf{X}^i_T) \right) ,$$

$$\text{s.t.} \quad \mathbf{X}^i_{t+1} = f^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) ,$$

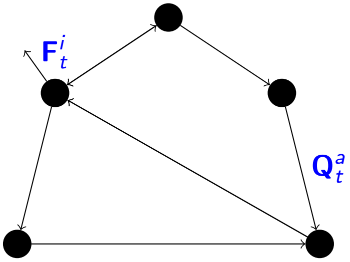
$$\mathbf{X}^i_t \in \mathcal{X}^{i,\mathrm{ad}}_t , \quad \mathbf{U}^i_t \in \mathcal{U}^{i,\mathrm{ad}}_t ,$$

$$\mathbf{U}^i_t \preceq \mathcal{F}_t ,$$

The last equation is the measurability constraint, where $\mathcal{F}_t$ is the σ-field generated by the noises $\{\mathbf{W}_\tau\}_{\tau=1\ldots t}$ up to time t.

Modeling exchanges between countries...

The grid is represented by a directed graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$. At each time $t \in [\![0, T-1]\!]$ we have:



- a flow $\mathbf{Q}_t^a$ through each arc a, inducing a cost $c_t^a(\mathbf{Q}_t^a)$, modeling the exchange between two countries
- a grid flow $\mathbf{F}_t^i$ at each node i, resulting from the balance equation

$$\mathbf{F}_t^i = \sum_{a \in input(i)} \mathbf{Q}_t^a - \sum_{b \in output(i)} \mathbf{Q}_t^b$$

...A transport problem decoupled in time

At each time step $t \in [\![0, T-1]\!]$, we define the transport cost as the sum of the cost of the flows $\mathbf{Q}_t^a$ through the arcs a of the grid:

$$J_{\mathfrak{T},t}[\mathbf{Q}_t] = \mathbb{E}\Big(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) \Big) ,$$

where the c_t^a's are easy to compute functions (say quadratic).

Kirchhof's law

The balance equation stating the conservation between $\mathbf{Q}_t$ and $\mathbf{F}_t$ rewrites in the following matrix form:

$$A\mathbf{Q}_t + \mathbf{F}_t = 0 ,$$

where A is the node-arc incidence matrix of the grid.

The overall production transport problem

The *production cost* aggregates the costs at all nodes i:

$$J_{\mathfrak{P}}[\mathbf{F}] = \sum_{i \in \mathcal{N}} J_{\mathfrak{P}}^{i}[\mathbf{F}^{i}] \ ,$$

and the *transport cost* J_T aggregates the costs at all time t:

$$J_{\mathfrak{T}}[\mathbf{Q}] = \sum_{t=0}^{T-1} J_{\mathfrak{T},t}[\mathbf{Q}_t] \ .$$

The compact production-transport problem formulation writes:

$$\min_{\mathbf{Q},\mathbf{F}} \quad J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}]$$

$$\text{s.t.} \quad A\mathbf{Q} + \mathbf{F} = 0 \ .$$

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Introducing Dual Approximate Dynamic Programming

The Decomposition/Coordination methods we want to deal with are iterative algorithms involving the following ingredients.

- Decompose the global problem in several subproblems of smaller size,
- Coordinate at each iteration the subproblems either with a price or an allocation,

$$A\mathbf{Q} + \underbrace{\mathbf{F}}_{\textit{allocation}} = 0 \quad \rightsquigarrow \underbrace{\boldsymbol{\lambda}}_{\textit{price}}$$

- Solve the subproblems using Dynamic Programming (when a state is available in the subproblem), taking into account the price or the allocation transmitted by the coordination.

Production subproblems induced by decomposition

The i-th production subproblem at iteration k formulates as follows.

- Price transmission case

$$\min_{\mathbf{X}^i,\mathbf{U}^i,\mathbf{F}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t^{(k)}, \mathbf{F}_t^i \rangle + K^i(\mathbf{X}_T^i) \Big) ,$$
$$\text{s.t. } \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^i, \mathbf{W}_{t+1}) ,$$
$$\mathbf{U}_t^i \preceq \mathcal{F}_t .$$

- Allocation transmission case

$$\min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^{i,(k)}, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \Big) ,$$
$$\text{s.t. } \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{F}_t^{i,(k)}, \mathbf{W}_{t+1}) ,$$
$$\mathbf{U}_t^i \preceq \mathcal{F}_t .$$

Approximating the subproblems

In both cases, the subproblems encompass a new "noise", that is, either a price multiplier $\boldsymbol{\lambda}_t^{(k)}$ or a flow allocation $\mathbf{F}_t^{i,(k)}$, which may be correlated in time. The white noise assumption fails.

Dynamic Programming cannot be used for solving the subproblems.

In order to overcome this difficulty, we use a trick that involves approximating the new noise (either $\boldsymbol{\lambda}_t^k$ or $\mathbf{F}_t^k$) by its conditional expectation w.r.t. a chosen random variable $\mathbf{Y}_t$.

Assume that the process $\mathbf{Y}$ has a given dynamics:

$$\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1}) .$$

If noises $\mathbf{W}_t$'s are time independent, then $(\mathbf{X}_t^i, \mathbf{Y}_t)$ is a valid state for the i-th subproblem and Dynamic Programming applies.[3]

[3]See [Barty et al, 2010] for further details.

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Price decomposition

The production and transport optimization problem writes

$$\min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \qquad \text{s.t.} \quad A\mathbf{Q} + \mathbf{F} = 0 \, . \tag{1}$$

The decomposition scheme consists in dualizing the constraint, and then approximating the multiplier $\boldsymbol{\lambda}$ by its conditional expectation w.r.t. $\mathbf{Y}$. This trick leads to the following problem

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \mathbb{E}(\boldsymbol{\lambda} \mid \mathbf{Y}) \, , A\mathbf{Q} + \mathbf{F} \right\rangle \, .$$

The last problem is equivalent to the two following problems.

1. Restricted dual problem

$$\max_{\boldsymbol{\lambda} \preceq \mathbf{Y}} \min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \boldsymbol{\lambda} \, , A\mathbf{Q} + \mathbf{F} \right\rangle \, ,$$

2. Relaxed primal problem ($\rightsquigarrow$ lower bound of (1))

$$\min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \qquad \text{s.t.} \quad \mathbb{E}\Big(A\mathbf{Q} + \mathbf{F} \,\Big|\, \mathbf{Y}\Big) = 0 \, .$$

A dual gradient-like algorithm

Applying the Uzawa algorithm to the problem

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{Q},\mathbf{F}} \; J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \mathbb{E}(\boldsymbol{\lambda} \mid \mathbf{Y}) \,, A\mathbf{Q} + \mathbf{F} \right\rangle \,,$$

leads to a decomposition between production and transport:

$$\mathbf{F}^{(k+1)} \in \arg\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + \left\langle \boldsymbol{\mu}^{(k)} \,, \mathbf{F} \right\rangle \,, \qquad \text{Production}$$

$$\mathbf{Q}^{(k+1)} \in \arg\min_{\mathbf{Q}} J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \boldsymbol{\mu}^{(k)} \,, A\mathbf{Q} \right\rangle \,, \qquad \text{Transport}$$

$$\boldsymbol{\mu}^{(k+1)} = \boldsymbol{\mu}^{(k)} + \rho \, \mathbb{E}\left(A\mathbf{Q}^{(k+1)} + \mathbf{F}^{(k+1)} \;\middle|\; \mathbf{Y} \right) \,, \qquad \text{Update}$$

where we use the notation $\boldsymbol{\mu}^{(k)} = \mathbb{E}\left(\boldsymbol{\lambda}^{(k)} \mid \mathbf{Y}\right)$.

Decomposing the transport problem

The transport subproblem

$$\min_{\mathbf{Q}} \; J_{\mathfrak{T}}[\mathbf{Q}] + \left\langle \boldsymbol{\mu}^{(k)} \, , A\mathbf{Q} \right\rangle \; ,$$

writes in a detailled manner

$$\min_{\mathbf{Q}} \sum_{t=0}^{T-1} \; \mathbb{E}\Big(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) + \left\langle \boldsymbol{\mu}_t^{(k)} \, , A\mathbf{Q}_t \right\rangle \Big) \; .$$

It is evidently decomposable in time and in space (arc by arc):

$$\min_{\mathbf{Q}_t^a} \; \mathbb{E}\Big(c_t^a(\mathbf{Q}_t^a) + \left\langle (A^\top \boldsymbol{\mu}_t^{(k)})^a \, , \mathbf{Q}_t^a \right\rangle \Big) \; ,$$

leading to subproblems extremely easy to solve.

Decomposing the production problem

The production subproblem

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + \left\langle \boldsymbol{\mu}^{(k)} , \mathbf{F} \right\rangle ,$$

decomposes node by node

$$\min_{\mathbf{F}^i} J^i_{\mathfrak{P}}[\mathbf{F}^i] + \left\langle \boldsymbol{\mu}^{i,(k)} , \mathbf{F}^i \right\rangle ,$$

that is,

$$\begin{aligned} \min_{\mathbf{X}^i,\mathbf{U}^i,\mathbf{F}^i} \ & \mathbb{E}\left(\sum_{t=0}^{T-1} \left(L^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) + \left\langle \boldsymbol{\mu}^{i,(k)}_t , \mathbf{F}^i_t \right\rangle \right) + K^i(\mathbf{X}^i_T) \right) \\ \text{s.t.} \ & \mathbf{X}^i_{t+1} = f^i_t(\mathbf{X}^i_t, \mathbf{U}^i_t, \mathbf{F}^i_t, \mathbf{W}_{t+1}) \\ & \mathbf{U}^i_t \preceq \mathcal{F}_t \ . \end{aligned}$$

Solving the production subproblems by DP

As $\mu_t^{(k)} \preceq \mathbf{Y}_t$, we write it as a functional $\mu_t^{(k)} = \phi_t^{(k)}(\mathbf{Y}_t)$.

We have assumed that

- the process $\mathbf{W}$ is a white noise,
- the process $\mathbf{Y}$ follows a dynamics $\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$,

so that Dynamic Programming applies for production subproblems:

$$
\begin{aligned}
V_T^i(x,y) &= K^i(x) \\
V_t(x,y) &= \min_{u,f} \mathbb{E}\Big(L_t^i(x,u,f,\mathbf{W}_{t+1}) \\
&\qquad + \big\langle \phi_t^{(k)}(y) \, , f \big\rangle + V_{t+1}^i(\mathbf{X}_{t+1}^i, \mathbf{Y}_{t+1}) \Big) \\
&\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(x,u,f,\mathbf{W}_{t+1}) \, , \\
&\qquad\ \ \mathbf{Y}_{t+1} = h_t(y, \mathbf{W}_{t+1}) \, .
\end{aligned}
$$

Resource allocation decomposition

Resource allocation decomposition applied to the problem

$$\min_{\mathbf{Q},\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] + J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad A\mathbf{Q} + \mathbf{F} = 0 \;, \tag{2}$$

consists in rewriting the constraint $A\mathbf{Q} + \mathbf{F} = 0$ by introducing a new variable $\mathbf{V}$ (the allocation), that is,

$$A\mathbf{Q} + \mathbf{V} = 0 \quad \text{and} \quad \mathbf{F} - \mathbf{V} = 0 \;.$$

Here the trick consists in limiting the measurability of variable $\mathbf{V}$, that is, $\mathbf{V} \preceq \mathbf{Y}$. The approximation leads to solve the following restricted primal problem ($\rightsquigarrow$ upper bound of (2))

$$\min_{\mathbf{V}\preceq\mathbf{Y}} \left(\min_{\mathbf{F}} \Big(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \Big) + \min_{\mathbf{Q}} \Big(J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad A\mathbf{Q} + \mathbf{V} = 0 \Big) \right) .$$

A primal gradient-like algorithm

Applying a gradient-like algorithm w.r.t. $\mathbf{V}$ to the problem

$$\min_{\mathbf{V} \preceq \mathbf{Y}} \Bigg(\min_{\mathbf{F}} \Big(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \Big) + \min_{\mathbf{Q}} \Big(J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad A\mathbf{Q} + \mathbf{V} = 0 \Big) \Bigg) ,$$

leads to a decomposition between production and transport:[4]

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V}^{(k)} = 0 \quad \rightsquigarrow \quad \boldsymbol{\lambda}^{(k+1)} \qquad \text{Production}$$

$$\min_{\mathbf{Q}} J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad A\mathbf{Q} + \mathbf{V}^{(k)} = 0 \quad \rightsquigarrow \quad \boldsymbol{\nu}^{(k+1)} \qquad \text{Transport}$$

$$\mathbf{V}^{(k+1)} = \mathrm{proj}_{\mathbf{V} \preceq \mathbf{Y}} \Big(\mathbf{V}^{(k)} + \rho \big(\boldsymbol{\lambda}^{(k+1)} - \boldsymbol{\nu}^{(k+1)} \big) \Big) \qquad \text{Update}$$

[4] Note that we must ensure at each iteration that $\mathbf{V}_t^{(k)} \in \mathrm{Im} A$.

Decomposing the transport problem

The transport subproblem

$$\min_{\mathbf{Q}} \; J_{\mathfrak{T}}[\mathbf{Q}] \quad \text{s.t.} \quad A\mathbf{Q} + \mathbf{V} = 0 \; ,$$

writes in a detailled manner

$$\min_{\mathbf{Q}} \sum_{t=0}^{T-1} \mathbb{E}\Big(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) \Big) \quad \text{s.t.} \quad A\mathbf{Q}_t + \mathbf{V}_t^{(k)} = 0 \;\; \forall t \; .$$

It is decomposable in time, but not in space:

$$\min_{\mathbf{Q}_t} \mathbb{E}\Big(\sum_{a \in \mathcal{A}} c_t^a(\mathbf{Q}_t^a) \Big) \quad \text{s.t.} \quad A\mathbf{Q}_t + \mathbf{V}_t^{(k)} = 0 \; .$$

The resulting subproblems are again easy to solve.

Decomposing the production problem

The production subproblem

$$\min_{\mathbf{F}} \Big(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \Big) \ ,$$

decomposes node by node

$$\min_{\mathbf{F}^i} J_{\mathfrak{P}}^i[\mathbf{F}^i] \quad \text{s.t.} \quad \mathbf{F}^i - \mathbf{V}^{i,(k)} = 0 \ ,$$

that is,

$$\min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{V}_t^{i,(k)}, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \Big) \ ,$$

$$\text{s.t.} \ \ \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{V}_t^{i,(k)}, \mathbf{W}_{t+1})$$

$$\mathbf{U}_t^i \preceq \mathcal{F}_t \ .$$

Solving the production subproblems by DP

As $\mathbf{V}_t^{i,(k)} \preceq \mathbf{Y}_t$, we write it as a functional $\mathbf{V}_t^{i,(k)} = \psi_t^{(k)}(\mathbf{Y}_t)$.

We have assumed that

- the process $\mathbf{W}$ is a white noise,
- the process $\mathbf{Y}$ follows a dynamics $\mathbf{Y}_{t+1} = h_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$,

so that Dynamic Programming applies for production subproblems:

$$V_T^i(x,y) = K^i(x)$$

$$V_t(x,y) = \min_u \mathbb{E}\Big(L_t^i(x,u,\psi_t^{(k)}(y),\mathbf{W}_{t+1}) + V_{t+1}^i(\mathbf{X}_{t+1}^i,\mathbf{Y}_{t+1})\Big)$$

$$\text{s.t.} \quad \mathbf{X}_{t+1}^i = f_t^i(x,u,\psi_t^{(k)}(y),\mathbf{W}_{t+1}) ,$$

$$\mathbf{Y}_{t+1} = h_t(y,\mathbf{W}_{t+1}) .$$

Interaction Prediction

As in resource allocation, we introduce a new variable $\mathbf{V}$ and rewrite the constraint $A\mathbf{Q} + \mathbf{F} = 0$ as

$$A\mathbf{Q} + \mathbf{V} = 0 \quad \text{and} \quad \mathbf{F} - \mathbf{V} = 0 \; .$$

We again limit the measurability of variable $\mathbf{V}$, that is, $\mathbf{V} \preceq \mathbf{Y}$. The interaction prediction is in that case a mix of price decomposition and resource allocation, aiming at solving:

$$\min_{\mathbf{V} \preceq \mathbf{Y}} \max_{\boldsymbol{\mu}} \left(\min_{\mathbf{F}} \Big(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \Big) + \min_{\mathbf{Q}} \Big(J_{\mathfrak{T}}[\mathbf{Q}] + \langle \boldsymbol{\mu} \, , A\mathbf{Q} + \mathbf{V} \rangle \Big) \right) .$$

Note that the constraint is partially handled (production problem) and partially dualized (transport problem).

A fixed-point algorithm

Applying a fixed-point algorithm w.r.t. $\mathbf{V}$ and $\boldsymbol{\mu}$ to the problem

$$\min_{\mathbf{V}\preceq\mathbf{Y}} \max_{\boldsymbol{\mu}} \left(\min_{\mathbf{F}} \Big(J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V} = 0 \Big) + \min_{\mathbf{Q}} \Big(J_{\mathfrak{T}}[\mathbf{Q}] + \langle \boldsymbol{\mu} \, , A\mathbf{Q} + \mathbf{V} \rangle \Big) \right) ,$$

leads to a decomposition between production and transport:

1. Solve the production and transport problems

$$\min_{\mathbf{F}} J_{\mathfrak{P}}[\mathbf{F}] \quad \text{s.t.} \quad \mathbf{F} - \mathbf{V}^{(k)} = 0 \quad \rightsquigarrow \quad \boldsymbol{\lambda}^{(k+1)} \quad \text{Production}$$

$$\min_{\mathbf{Q}} J_{\mathfrak{T}}[\mathbf{Q}] + \langle \boldsymbol{\mu}^{(k)} \, , A\mathbf{Q} \rangle \quad \rightsquigarrow \quad \mathbf{Q}^{(k+1)} \quad \text{Transport}$$

2. Update the allocation and the multiplier:

$$\mathbf{V}^{(k+1)} = A\mathbf{Q}^{(k+1)} \quad , \quad \boldsymbol{\mu}^{(k+1)} = \boldsymbol{\lambda}^{(k+1)} \, .$$

Decomposing the production and the transport problems

In prediction decomposition, the production subproblem is solved in the same way as in ressource allocation, whereas the transport subproblem is solved in the same way as in price decomposition.

All that has been seen above therefore applies:

- the production subproblem decomposes node by node:

$$\min_{\mathbf{F}^i} J^i_{\mathfrak{P}}[\mathbf{F}^i] \quad \text{s.t.} \quad \mathbf{F}^i - \mathbf{V}^{i,(k)} = 0 \ ,$$

  (Dynamic Programming applies)

- the transport subproblem decomposes in time and in space:

$$\min_{\mathbf{Q}_t} \mathbb{E}\Big(c^a_t(\mathbf{Q}^a_t) + \langle (A^T \boldsymbol{\mu}^{(k)}_t)^a \, , \mathbf{Q}^a_t \rangle \Big) \ ,$$

  (easy to solve subproblems)

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We want to benchmark these three methods, with:

- A numerical comparison, applying these algorithms to manage the European grid.
- A theoretical comparison: knowing that

$$\mathfrak{J}^{price} \leq \mathfrak{J}^{\sharp} \leq \mathfrak{J}^{resource}$$

do we have

$$\mathfrak{J}^{price} \leq \mathfrak{J}^{prediction} \leq \mathfrak{J}^{resource} \quad ?$$

- An implementation of other decomposition methods, such as augmented Lagrangian based methods.

K. Barty, P. Carpentier, G. Cohen and P. Girardeau.
Price decomposition in large-scale stochastic optimal control.
arXiv, math.OC 1012.2092, 2010.

P. Carpentier, J.-P. Chancelier, V. Leclère, F. Pacaud.
Stochastic decomposition applied to large-scale hydro valleys management.
Working Paper, 2017.

G. Cohen.
Auxiliary Problem Principle and Decomposition of Optimization Problems.
Journal of Optimization Theory and Applications 32, 277-305, 1980.

P. Girardeau.
Résolution de grands problèmes en optimisation stochastique dynamique.
Thèse de doctorat, Université Paris-Est, 2010.

V. Leclère.
Contributions aux méthodes de décomposition en optimisation stochastique.
Thèse de doctorat, Université Paris-Est, 2014.

A. Lenoir and P. Mahey.
A survey of monotone operator splitting methods and decomposition of convex programs.
RAIRO Operations Research 51, 17-41, 2017.